

Some Thermodynamic Aspects of Pure Glue, Fuzzy Bags and Gauge/String Duality

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Abstract

The thermodynamic properties of a $SU(3)$ gauge theory without quarks are calculated using a string formulation for $1.2 T_c \leq T \leq 3 T_c$. The results are in good agreement with the lattice data. We also comment on $SU(N)$ gauge theories.

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1 Introduction

High energy nucleus-nucleus collisions provide the means of creating nuclear matter in conditions of extreme temperature and density. In particular, the system undergoes a transition from a state of nucleons containing bound quarks and gluons to a state of deconfined quarks and gluons. This state was originally given the name Quark Gluon Plasma. However, the results at RHIC indicate that instead of behaving like a weakly coupled gas of free quarks and gluons, the matter created in heavy ion collisions behaves like a strongly coupled liquid.¹ Thus, there is a need for new approaches to strongly coupled gauge theories.

Until recently, the lattice formulation was a unique theoretical tool to deal with strongly coupled gauge theories. The subject has taken an interesting turn with Maldacena duality [2]. One of the implications is that it resumed interest in finding a string description of strong interactions. Although the original proposal was for conformal theories, various modifications have been found that produce gauge/string duals with a mass gap, confinement, and supersymmetry breaking [3].

In this paper we address some issues of thermodynamics of $SU(3)$ pure gauge theory in a dual formulation. Clearly, finding the dual from first principles of string theory is beyond of our ability. Instead, we attempt the inverse problem and use our knowledge of some phenomenologically successful five-dimensional models of AdS/QCD.

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¹For recent reviews, see [1].

Before proceeding to the detailed analysis, let us set the basic framework. We consider the following ansatz for the 10-dimensional background geometry which turns out to be applicable for the temperature range $1.2 T_c \leq T \leq 3 T_c$ ²

$$ds^2 = \frac{R^2}{z^2} H (f dt^2 + d\vec{x}^2 + f^{-1} dz^2) + H^{-1} g_{ab} d\omega^a d\omega^b, \quad f = 1 - \left(\frac{z}{z_T}\right)^4, \quad H = e^{\frac{1}{2} c z^2}. \quad (1)$$

Here $z_T = 1/\pi T$. It is a deformed product of the Euclidean AdS_5 black hole and a 5-dimensional sphere (compact space) whose coordinates are ω^a . The deformation is due to a z -dependent factor H . Such a deformation is crucial for breaking conformal invariance of the original supergravity solution and introducing Λ_{QCD} .

Apart from the language of 10-dimensional string theory, there is a more phenomenological way to attack QCD. This approach called AdS/QCD deals with a five-dimensional effective description and tries to fit it to QCD as much as possible. For our model, its AdS/QCD cousin can be obtained by discarding the compact space in (1).

At $T = 0$, then what we get is the slightly deformed AdS_5 metric. Such a deformation is notable. The point is that in this background linearized Yang-Mills equations are effectively reduced to a Laguerre differential equation. As a result, the spectrum turns out to be like that of the linear Regge models [4, 5]. This fact allows one to fix the value of c from the ρ meson trajectory. It is of order [5]

$$c \approx 0.9 \text{ GeV}^2. \quad (2)$$

We will assume that the value of c is universal and is therefore valid for the world without quarks too. In addition, this AdS/QCD model provides the phenomenologically acceptable heavy quark potentials as well as the value of the gluon condensate [6, 7].

At finite T , the model provides the spatial string tension of pure gauge theory [8]. The agreement with the lattice data is very good for temperatures lower than $2.5\text{-}3 T_c$. Due to this reason we set the upper bound on T in (1). Moreover, the model describes in a qualitative way a heavy quark-antiquark pair and the expectation value of the Polyakov loop [9].

Thus, there are reasons to believe that the model (1) is a good approximation for a string dual to a pure gauge theory.

2 Thermodynamics

2.1 The Entropy Density

One of the bedrocks of gauge/string (gravity) duality is a conjecture that the entropy of gauge theories is equal to the Bekenstein-Hawking entropy of their string (gravity) duals [3]. As known, the Bekenstein-Hawking entropy is proportional to an (8-dimensional) area of the horizon. We can now ask whether the five-dimensional framework (AdS/QCD) is an adequate approximation at this point. In general, the answer is no. There is a contribution from the compact space that might be relevant.

²The lower limit is chosen to keep the system out of the critical regime. As we will discuss below, the upper limit is determined by consistency rather than perturbation theory.

The metric (1) has the horizon at $z = z_{\text{T}}$. Therefore, the temperature dependence of the entropy density is³

$$s(T) = s_0 T^3 \exp \left\{ -\frac{1}{2} \frac{T_c^2}{T^2} \right\}, \quad (3)$$

where s_0 is a factor independent of temperature. In this formula T_c is given by⁴

$$T_c = \frac{1}{\pi} \sqrt{c}. \quad (4)$$

It follows from (3) that the entropy density can be represented as a series in powers of $\frac{1}{T^2}$ with the leading T^3 term

$$s(T) = s_0 T^3 \sum_{n=0}^{\infty} a_n \tau^n, \quad \tau = \frac{T_c^2}{T^2}, \quad (5)$$

where $a_n = \frac{(-)^n}{2^n n!}$.

For future use, we define the truncated model by keeping the two leading terms in (5). We have

$$s_{\text{tr}}(T) = s_0 T^3 \left(1 - \frac{1}{2} \tau \right). \quad (6)$$

2.2 The Pressure

2.2.1 Fuzzy Bags

Recently, it has been suggested by Pisarski that for the temperature range $T_{\text{max}} < T < T_{\text{pert}}$ the pressure in QCD with quarks is given by a series in powers of $\frac{1}{T^2}$ times the ideal T^4 term [10]. Explicitly,

$$p_{\text{QCD}}(T) \approx f_{\text{pert}} T^4 - B_{\text{fuzzy}} T^2 - B_{\text{MIT}} + \dots \quad (7)$$

It was called a fuzzy bag model for the pressure. So, B_{MIT} stands for the MIT bag constant. T_{max} is close to a critical temperature T_c (or some approximate " T_c'' " for a crossover). A small difference between T_c and T_{max} may vary with the model. T_{pert} is set by perturbation theory such that it is applicable only for temperatures higher than T_{pert} .

For pure glue, Pisarski argued, based on lattice simulations of [11], that (7) reduces to

$$p(T) \approx f_{\text{pert}} (T^4 - T_c^2 T^2). \quad (8)$$

This means that $B_{\text{fuzzy}} = f_{\text{pert}} T_c^2$ and B_{MIT} is much smaller than the first two terms. So, the pressure is a sum of just two pieces. Note that an important consequence of (8) is that the pressure (nearly) vanishes at $T = T_c$.

³We take a constant dilaton.

⁴In the following section we will see that T_c can be thought as a critical temperature.

2.2.2 String Dual

Given the entropy density as a function of T , in the homogeneous case one can find the temperature dependence of the pressure by integrating $\frac{dp}{dT} = s$.⁵ From (5), we get

$$p(T) = \frac{1}{4}s_0 T^4 \left(1 - \tau - \frac{1}{4}\tau^2 \ln \tau - b\tau^2 + \sum_{n=3}^{\infty} b_n \tau^n \right), \quad (9)$$

where b is an integration constant and $b_n = \frac{2a_n}{2-n}$.

The final topic to be considered here is whether the proposal of Pisarski is reasonable in the model under consideration. The two leading terms in (9) look similar to those of (8). So, we find that the critical temperature is given by T_c . A simple estimate then gives⁶

$$T_c \approx 300 \text{ MeV}. \quad (10)$$

In SU(3) pure gauge theory the critical temperature is of order 270 MeV. So, the agreement is not bad at this point.

Let us now use $p(T_c) = 0$ to determine the integration constant. As a result, we have

$$b = \sum_{n=3}^{\infty} b_n \approx 0.039. \quad (11)$$

The value of b is indeed small compared to the coefficients in front of the two leading terms. Thus, the agreement is very satisfactory at this point.

To complete the picture, we present the results of numerical calculations. We split the series (9) into two pieces, the first containing the two leading terms, and the second presenting the rest. Then we define⁷

$$p_1(T) = 1 - \tau, \quad p_2(T) = -\frac{1}{4}\tau^2 \ln \tau - b\tau^2 + \sum_{n=3}^{\infty} b_n \tau^n. \quad (12)$$

For the sake of simplicity, we have omitted the overall factor $\frac{1}{4}s_0 T^4$. The values of p_1 and p_2 can be read off of Fig.1. We see that at $T \approx 1.2T_c$ the value of p_2 is one order of magnitude

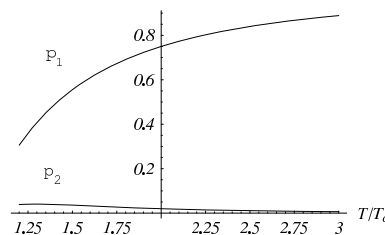


Figure 1: Values of p_1 and p_2 versus the ratio $\frac{T}{T_c}$.

⁵In what follows, we consider the homogeneous case.

⁶We use (2) for this estimate.

⁷Note that the truncated model (5) can be derived from p_1 times $\frac{1}{4}s_0 T^4$.

smaller than that of p_1 . Above $1.2T_c$ the value of p_1 increases, while p_2 decreases and becomes negligible for $T \gtrsim 2T_c$. Thus, $p_1(T)$ provides a reliable approximation whose error is less than 10% for the pressure.

In sum, the truncated model which is equivalent to the proposal of Pisarski is valid with accuracy better than 10%.

2.3 The Speed of Sound

Having derived the entropy density, we can easily obtain the speed of sound. For the model of interest, we have

$$C_s^2(T) = \frac{s}{Ts'} = \frac{1}{3} \left(1 + \frac{1}{3}\tau\right)^{-1}. \quad (13)$$

For completeness, we also present the result obtained for the truncated model (6). In this case (13) is replaced by

$$C_s^2(T) = \frac{1}{3} \left(1 - \frac{1}{2}\tau\right) \left(1 - \frac{1}{6}\tau\right)^{-1}. \quad (14)$$

Note that C_s is independent of s_0 . Thus, we do not have any free fitting parameter at this point.

We close the discussion of the speed of sound by comparing the results with those of lattice simulations.⁸ The curves are shown in Fig.2.

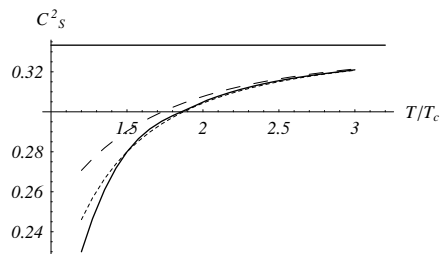


Figure 2: The square of the speed of sound versus $\frac{T}{T_c}$. The upper and lower dashed curves correspond to (13) and (14), respectively. The solid curve represents the result of the extrapolation to the continuum limit for lattice simulations [11]. The solid horizontal line is the usual AdS/CFT result with the value $\frac{1}{3}$.

From the Figure we can see that the model of interest is in very good agreement with the lattice for $T \gtrsim 1.7T_c$, while near $1.2T_c$ the discrepancy is of order 15 percent. The agreement between the truncated model and the lattice is spectacular. The maximum discrepancy occurred at $T = 1.2T_c$ is of order 6 percent.

2.4 The Gluon Condensate at Finite Temperature

We will next describe the gluon condensate at finite temperature.⁹ It is obtained from the trace anomaly of the energy-momentum tensor [13]. We have

⁸The recent data of [12] have large error bars. So, it is impossible to say how precisely the results fit.

⁹Although the literature on the gluon condensate is very vast, to our knowledge, there are no reliable results for the temperature range $1.2T_c \leq T \leq 3T_c$ except those coming from lattice simulations.

$$G_2(T) = G_2 + 4p - Ts, \quad (15)$$

where G_2 is the condensate at zero temperature.

Unlike the speed of sound, the condensate depends on the parameter s_0 . There are two different ways to fix its value which fortunately yield very similar results. The first is to fit the interaction measure $(\epsilon - 3p)/T^4$ as it follows from (5) and (9) to the lattice data of [11] at some normalization point T_n . As result, we get

$$s_0 = 6.8 \pm 0.3. \quad (16)$$

At first glance it may seem curious that the result is almost independent of the normalization point. As we will see in a moment, this is indeed the case.

The second is to match the coefficient in front of the T^4 term in (9) with that of the bag model [14]. For $SU(N)$ (pure) gauge theory, the latter is simply $\frac{N^2-1}{45}\pi^2$. At $N = 3$, we find

$$s_0 = \frac{32}{45}\pi^2 \approx 7.0 \quad (17)$$

that is really the same as (16).

Having determined the value of s_0 , we can now write down the expression for the condensate. Combining (15), and (5) and (9), we get

$$G_2(T) = -s_0 T^4 \left(\frac{1}{2}\tau + \frac{1}{4}\tau^2 \ln \tau + g\tau^2 + \sum_{n=3}^{\infty} (a_n - b_n)\tau^n \right), \quad (18)$$

where $g = \frac{1}{8} + b - \frac{k}{s_0}$. Note that the condensate at zero temperature $G_2 = kT_c^4$ has been included in the τ^2 term. For the background geometry (1), the estimate of [7] gives $k \approx 1.20$. Interestingly, the value of g turns out to be small. For $s_0 = 6.8$ it is of order -0.01 .

In Fig.3 we have plotted the gluon condensate in units of T_c^4 as a function of the ratio $\frac{T}{T_c}$.

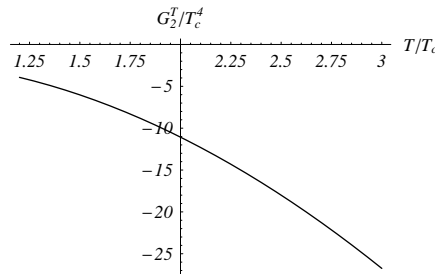


Figure 3: The gluon condensate in units of T_c^4 versus $\frac{T}{T_c}$. Here $s_0 = 6.8$.

We conclude the discussion with a couple of comments:

- (i) The expression for the gluon condensate is cumbersome and difficult of any practical use. We should therefore seek a simpler (nearly equal) expression. What we already know is that the truncated model is a good approximation for the pressure and the speed of sound. So, it is

reasonable to use this option. We can check it by the same arguments that we used in the case of the pressure. To this end, we split the series (18) into two pieces and define

$$g_1(T) = -\frac{1}{2}\tau, \quad g_2(T) = -\frac{1}{4}\tau^2 \ln \tau - g\tau^2 + \sum_{n=3}^{\infty} (b_n - a_n)\tau^n. \quad (19)$$

For simplicity, we have omitted the overall factor $s_0 T^4$.

The values of g_1 and g_2 can be read off of Fig.4. We see that the value of g_2 is approximately

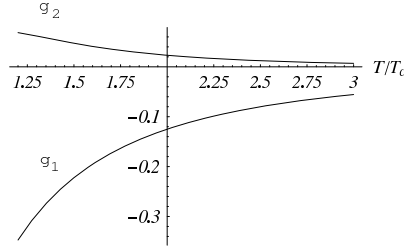


Figure 4: Values of g_1 and g_2 versus the ratio $\frac{T}{T_c}$.

15% of g_1 . Thus, in the temperature range under consideration we may approximate the infinite series (18) by g_1 . Finally, the gluon condensate takes the form predicted by the truncated model

$$G_2(T) \approx -\frac{s_0}{2} T_c^2 T^2. \quad (20)$$

(ii) Using (5) and (9), one can easily find the expression for the interaction measure. It is

$$\frac{\epsilon - 3p}{T^4} = s_0 \left(\frac{1}{2}\tau + \frac{1}{4}\tau^2 \ln \tau + \left(b + \frac{1}{8}\right)\tau^2 + \sum_{n=3}^{\infty} (a_n - b_n)\tau^n \right). \quad (21)$$

The truncated model provides a simpler expression of the measure

$$\frac{\epsilon - 3p}{T^4} = \frac{s_0}{2} \tau, \quad (22)$$

as expected.

In Fig.5 we have plotted the interaction measure as a function of the ratio $\frac{T}{T_c}$. As can be seen from the Figure, the agreement with the lattice data is very satisfactory. An important observation is that varying s_0 over the range (16) has a little effect.

3 Concluding Comments

(i) The model we have proposed predicts the entropy density as a series in $\frac{1}{T^2}$. It differs from the proposal of Pisarski [10] by having a term $\ln T$ in the pressure. However, in the pure glue case the $\ln T$ term turns out to be subdominant.

(ii) Interestingly enough, the spatial string tension calculated within the AdS/QCD cousin of (1) is given by [8]

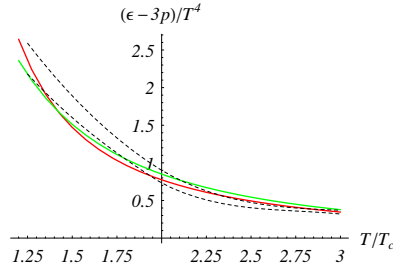


Figure 5: The interaction measure $(\epsilon - 3p)/T^4$ versus $\frac{T}{T_c}$. The red and green curves correspond to (21) and (22), respectively. The lattice data of [11] lie between the two dashed lines. Here $s_0 = 6.8$.

$$\sigma_s = \sigma \frac{e^{\tau-1}}{\tau}, \quad (23)$$

where σ is the string tension at zero temperature. Then, from (23), we see that the spatial tension can be written as a series in powers of $\frac{1}{T^2}$ times T^2 . Note that unlike the cases considered in section 2 the first two terms of the series do not provide a reasonable approximation.

(iii) Can one think of the model (1) as a string dual to a $SU(N)$ pure gauge theory? We will be exploring the consequences of assuming that the pressure vanishes at $T = T_c$.¹⁰

This assumption leads to the same expression for the pressure as (9) with b defined by (11). The overall constant s_0 is fixed from the T^4 term. Fitting the bag model, we have

$$s_0 = \frac{4\pi^2}{45} (N^2 - 1). \quad (24)$$

Clearly, the analysis of section 2 is not sensitive to N . So, the conclusion we draw is that the truncated model which is equivalent to (8) is valid with accuracy better than 10%.

Moreover, we can obtain a formula for the pressure normalized by the leading term $p_0 = \frac{1}{4}s_0T^4$. It is

$$\frac{p}{p_0}(\tau) = \left(1 - \tau - \frac{1}{4}\tau^2 \ln \tau - b\tau^2 + \sum_{n=3}^{\infty} b_n \tau^n\right). \quad (25)$$

Thus our model predicts that the ratio is a function of $\frac{T}{T_c}$.¹¹ It does not explicitly depend on N . At this point it is worth noting that in addition to $N = 3$ the prediction is also supported by lattice simulations for $N = 4$ and $N = 8$ [15].

(iv) We can gain some understanding of the N dependence of a parameter $\mathfrak{g} = \frac{R^2}{\alpha'}$. Here α' is the usual string parameter coming from the Nambu-Goto action.

The lattice data are well fitted by [16]

$$\frac{T_c}{\sqrt{\sigma}} = 0.596 + \frac{0.453}{N^2}, \quad (26)$$

¹⁰The parameter c is now dependent of N , so (2) is not applicable.

¹¹Strictly speaking, it is a function of $\frac{T^2}{T_c^2}$.

where σ is the string tension at zero temperature. For the AdS/QCD cousin of (1) it is given by [6]

$$\sigma = \mathfrak{g} \frac{e}{4\pi} c. \quad (27)$$

Combining (26), and (4) and (27), we learn

$$\mathfrak{g} = \frac{4}{\pi e} \left(0.596 + \frac{0.453}{N^2} \right)^{-2}. \quad (28)$$

A simple algebra shows that \mathfrak{g} is a slowly varying function of N . It takes values between 0.93 at $N = 2$ and 1.32 at $N = \infty$.

For $N = 3$, \mathfrak{g} is approximately equal to 1.12. It is interesting to compare this value with the estimate of [6]. The latter was made by using the Cornell potential. The result is $\mathfrak{g} \approx 0.94$. The estimates are relatively close. This might be a hint that \mathfrak{g} is also a slowly varying function of a number of quarks.

Acknowledgments

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